

Homework #5

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1 Number 1: Exercise 7.7.2

How does the context-free pumping lemma fail for each of the following languages?

a. $L = \{00, 11\}$

It is not possible to pick an arbitrary n because $\{w \mid |w| \neq 2; w \in L\} = \emptyset$

b. $L = \{0^n 1^n \mid n \geq 1\}$

With:

$$w = uvxyz \quad (1)$$

$$|w| \geq 2 \quad (2)$$

$$u = 0^{n_u} \ni n_u = \frac{|w|}{2} - 1 \quad (3)$$

$$v = 0 \quad (4)$$

$$x = \epsilon \quad (5)$$

$$y = 1 \quad (6)$$

$$z = 1^{n_z} \ni n_z = \frac{|w|}{2} - 1 \quad (7)$$

$$w_i = uv^i xy^i z \in L \quad (8)$$

The pumping lemma for context free languages is satisfied \therefore L is context-free.

c. The set of all palindromes where $\Sigma = \{0, 1\}$. That is:

$$L = \{waw^R \mid w \in \Sigma^*; a \in \Sigma \cup \{\epsilon\}\} \quad (9)$$

With:

$$w = uvxyz \quad (10)$$

$$|w| \geq 3 \quad (11)$$

$$u = \epsilon \quad (12)$$

$$v = a \ni a \in \Sigma \quad (13)$$

$$x \in L \quad (14)$$

$$y = a \quad (15)$$

$$z = \epsilon \quad (16)$$

$$w_i = uv^i xy^i z \in L \quad (17)$$

The pumping lemma for context free languages is satisfied \therefore L is context-free.

2 Number 2: Exercise 7.3.1

Show the context free languages are closed under $init \ni$

$$init(L) = \{w | \exists x \ni wx \in L\} \quad (18)$$

3 Number 3: Exercise 7.3.2

Consider the languages:

$$L_1 = \{a^n b^{2n} c^m | n, m \in \mathbb{N}\} \quad (19)$$

$$L_2 = \{a^n b^m c^{2m} | n, m \in \mathbb{N}\} \quad (20)$$

a. Show that L_1 and L_2 are context-free using grammars:

$$S_1 \rightarrow AC$$

$$A \rightarrow aAbb | \epsilon$$

$$C \rightarrow cC | \epsilon$$

$$S_2 \rightarrow AB$$

$$A \rightarrow aA | \epsilon$$

$$C \rightarrow bBcc | \epsilon$$

State	0	1	X	Y	B
$> q_0$	(q_1, X, R)	-	-	(q_3, Y, R)	-
q_1	$(q_1, 0, R)$	(q_2, Y, L)	-	(q_1, Y, R)	-
q_2	$(q_2, 0, L)$	-	(q_0, X, R)	(q_2, Y, L)	-
q_3	-	-	-	(q_3, Y, R)	(q_4, B, R)
q_4^*	-	-	-	-	-

Table 1: Turing machine to accept $\{0^n 1^n | n \geq 1\}$

b. Is $L_1 \cap L_2$ context-free? Why?

$$L_1 \cap L_2 = \{a^i b^j c^k | 2i = j; 2j = k; i, j, k \in \mathbb{N}\} = \{a^n b^{2n} c^{4n} | n \in \mathbb{N}\}$$

4 Number 4: Exercise 7.4.2

Give linear time algorithms answering the following questions:

a. Which symbols in a context-free grammar appear in some sentential form?

b. Which symbols in a context-free grammar are nullable?

5 Number 5: Exercise 8.2.1

For the Turing machine in Table ??:

Give the instantaneous descriptions of the following strings:

a. 00

$$\begin{aligned} q_0 \overset{\dagger}{\underset{0}{|}} &\vdash q_1 X \overset{\dagger}{\underset{0}{|}} \\ &\vdash q_2 X 0 \overset{\dagger}{\underset{|}{|}} \end{aligned}$$

b. 00011

$$\begin{aligned}
q_0 \frac{\dagger}{0} 0011 &\vdash q_1 X \frac{\dagger}{0} 011 \\
&\vdash q_1 X 0 \frac{\dagger}{0} 11 \\
&\vdash q_1 X 00 \frac{\dagger}{1} 1 \\
&\vdash q_2 X 0 \frac{\dagger}{0} Y 1 \\
&\vdash q_2 X \frac{\dagger}{0} 0 Y 1 \\
&\vdash q_2 \frac{\dagger}{X} 00 Y 1 \\
&\vdash q_0 X \frac{\dagger}{0} 0 Y 1 \\
&\vdash q_1 X X \frac{\dagger}{0} Y 1 \\
&\vdash q_1 X X 0 \frac{\dagger}{Y} 1 \\
&\vdash q_1 X X 0 Y \frac{\dagger}{1} \\
&\vdash q_2 X X 0 \frac{\dagger}{Y} Y \\
&\vdash q_2 X X \frac{\dagger}{0} Y Y \\
&\vdash q_2 X \frac{\dagger}{X} 0 Y Y \\
&\vdash q_0 X X \frac{\dagger}{0} Y Y \\
&\vdash q_1 X X X \frac{\dagger}{Y} Y \\
&\vdash q_1 X X X Y \frac{\dagger}{Y} \\
&\vdash q_1 X X X Y Y \frac{\dagger}{Y}
\end{aligned}$$

c. 00111

$$\begin{aligned}
q_0 \overset{\dagger}{0}0111 &\vdash q_1 X \overset{\dagger}{0}111 \\
&\vdash q_1 X 0 \overset{\dagger}{1}11 \\
&\vdash q_2 X \overset{\dagger}{0}Y11 \\
&\vdash q_2 \overset{\dagger}{X}0Y11 \\
&\vdash q_0 X \overset{\dagger}{0}Y11 \\
&\vdash q_1 X X \overset{\dagger}{Y}11 \\
&\vdash q_1 X X Y \overset{\dagger}{1}1 \\
&\vdash q_2 X X \overset{\dagger}{Y}Y1 \\
&\vdash q_2 X \overset{\dagger}{X}Y Y1 \\
&\vdash q_0 X X \overset{\dagger}{Y}Y1 \\
&\vdash q_3 X X Y \overset{\dagger}{Y}1 \\
&\vdash q_3 X X Y Y \overset{\dagger}{1}
\end{aligned}$$

6 Number 6: Exercise 8.2.2

Design Turing machines for the following languages:

a. $\{w \mid \text{count}_0(w) = \text{count}_1(w)\}$

The JFlap file for this Turing machine is available at:
<http://odin.himinbi.org/classes/csc445/8.2.2.a.TM>

b. $\{a^n b^n c^n \mid n \in \mathbb{N}\}$

The JFlap file for this Turing machine is available at:
<http://odin.himinbi.org/classes/csc445/8.2.2.a.TM>

c. $\{ww^R \mid w \in \Sigma^*; \Sigma = \{0, 1\}\}$

The JFlap file for this Turing machine is available at:
<http://odin.himinbi.org/classes/csc445/8.2.2.a.TM>

7 Number 7: Exercise 8.2.3

Design a Turing machine which adds one to a given number in binary.

- a. What are the transitions in this machine and the purpose of each?

The JFlap file for this Turing machine is available at:
<http://odin.himinbi.org/classes/csc445/8.2.3.TM>

q_0 → Start and move to the left side of the string

q_1 → Move back to the left over 1's changing them to 0's

q_2^* → On a 0 on B, that digit is the end of the ripple and will become a 1, leaving the rest of the string unaltered

- b. Show the sequence of instantaneous descriptions for the string 111:

$$\begin{aligned}
 q_0 \overset{\dagger}{\underset{1}{|}} 11 &\vdash q_0 1 \overset{\dagger}{\underset{1}{|}} 1 \\
 &\vdash q_0 11 \overset{\dagger}{\underset{1}{|}} \\
 &\vdash q_0 111 \overset{\dagger}{\underset{1}{|}} \\
 &\vdash q_1 11 \overset{\dagger}{\underset{1}{|}} \\
 &\vdash q_1 1 \overset{\dagger}{\underset{1}{|}} 0 \\
 &\vdash q_1 \overset{\dagger}{\underset{1}{|}} 00 \\
 &\vdash q_1 \overset{\dagger}{\underset{1}{|}} 000 \\
 &\vdash q_2 1 \overset{\dagger}{\underset{0}{|}} 00
 \end{aligned}$$